[09-03-24-AT-11]

Law of sines

A *Solve each triangle given the indicated measures of angles and sides.*

1. $\beta = 43°, \quad \gamma = 36°, \quad a = 92$ cm

2. $\alpha = 122°, \quad \gamma = 18°, \quad b = 12$ mm

3. $\beta = 27.5°, \quad \gamma = 54.5°, \quad a = 9.27$ mm

4. $\alpha = 118.3°, \quad \gamma = 12.2°, \quad b = 17.3$ km

5. $\alpha = 122.7°, \quad \beta = 34.4°, \quad b = 18.3$ cm

6. $\alpha = 67.7°, \quad \beta = 54.2°, \quad b = 123$ ft

7. $\beta = 12°40', \quad \gamma = 100°0', \quad b = 13.1$ km

8. $\alpha = 73°50', \quad \beta = 51°40', \quad a = 36.6$ mm

B *In Problems 9–16, determine whether the information in each problem allows you to construct 0, 1, or 2 triangles. Do not solve the triangle. Explain which case in Table 2 applies.*

9. $a = 3$ ft, $\quad b = 6$ ft, $\quad \alpha = 30°$

10. $a = 2$ in., $\quad b = 4$ in., $\quad \alpha = 30°$

11. $a = 8$ ft, $\quad b = 6$ ft, $\quad \alpha = 30°$

12. $a = 6$ in., $\quad b = 4$ in., $\quad \alpha = 30°$

13. $a = 2$ ft, $\quad b = 6$ ft, $\quad \alpha = 30°$

14. $a = 1$ in., $\quad b = 4$ in., $\quad \alpha = 30°$

15. $a = 5$ ft, $\quad b = 6$ ft, $\quad \alpha = 30°$

16. $a = 3$ in., $\quad b = 4$ in., $\quad \alpha = 30°$

In Problems 17–24, solve each triangle. If a problem has no solution, say so. If a problem involves two triangles, solve both unless stated to the contrary.

17. $\alpha = 27.5°, \quad a = 15.0$ mm, $\quad b = 36.4$ mm

18. $\alpha = 47.7°, \quad a = 8.5$ ft, $\quad b = 12.5$ ft

19. $\alpha = 135°20', \quad a = 14.6$ m, $\quad b = 18.3$ m

20. $\alpha = 122°40', \quad a = 105$ mi, $\quad b = 152$ mi

21. $\beta = 33°50', \quad a = 673$ ft, $\quad b = 1{,}240$ ft

22. $\beta = 29°30', \quad a = 43.2$ in., $\quad b = 56.5$ in.

23. $\beta = 27.3°, \quad a = 244$ ft, $\quad b = 135$ ft

24. $\beta = 38.9°, \quad a = 42.7$ cm, $\quad b = 30.0$ cm

C **25.** **Mollweide's equation,**

$$(a - b)\cos\frac{\gamma}{2} = c\sin\frac{\alpha - \beta}{2}$$

is often used to check the final solution of a triangle since all six parts of a triangle are involved in the equation. If, after substitution, the left side does not equal the right side, then an error has been made in solving a triangle. Use this equation to check Problem 1 to two decimal places. (Remember that rounding may not produce exact equality, but the left and right sides of the equation should be close.)

26. Use Mollweide's equation (see Problem 25) to check Problem 3 to two decimal places.

27. Use the law of sines and suitable identities to show that for any triangle,

$$\frac{a - b}{a + b} = \frac{\tan\dfrac{\alpha - \beta}{2}}{\tan\dfrac{\alpha + \beta}{2}}$$

28. Verify (to three decimal places) the formula in Problem 27 with values from Problem 1 and its solution.

29. Let $\beta = 46.8°$ and $a = 66.8$ yd. Determine a value k so that if $0 < b < k$, there is no solution; if $b = k$, there is one solution; and if $k < b < a$, there are two solutions.

30. Let $\beta = 36.6°$ and $b = 12.2$ m. Determine a value k so that if $0 < b < k$, there is no solution; if $b = k$, there is one solution; and if $k < b < a$, there are two solutions.

Applications

31. **Surveying** To determine the distance across the Grand Canyon in Arizona, a 1.00 mi baseline, AB, is established along the southern rim of the canyon. Sightings are then made from the ends (A and B) of the baseline to a point C across the canyon (see the figure). Find the distance from A to C if $\angle BAC = 118.1°$ and $\angle ABC = 58.1°$.

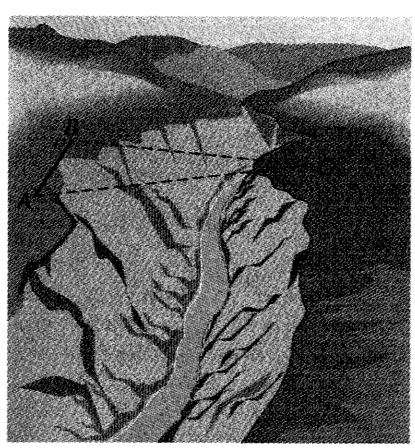

Figure for 31

Exercise 6.1

1. $\alpha = 101°$, $b = 64$ cm, $c = 55$ cm
3. $\alpha = 98.0°$, $b = 4.32$ mm, $c = 7.62$ mm
5. $\gamma = 22.9°$, $a = 27.3$ cm, $c = 12.6$ cm
7. $\alpha = 67°20'$, $a = 55.1$ km, $c = 58.8$ km
9. 1 triangle; case (b), where α is acute and $a = 3 = h$
11. 1 triangle; case (d), where α is acute and $a \geq b$ ($a = 8, b = 6$)
13. 0 triangles; case (a), where α is acute and $0 < a < h$ ($a = 2, h = 3$)
15. 2 triangles; case (c), where α is acute and $h < a < b$ ($h = 3, a = 5, b = 6$)
17. No solution
19. No solution
21. $\alpha = 17°40'$, $\gamma = 128°30'$, $c = 1{,}740$ ft
23. Triangle 1: $\alpha = 124.0°$, $\gamma = 28.7°$, $c = 141$ ft;
Triangle 2: $\alpha' = 56.0°$, $\gamma' = 96.7°$, $c' = 292$ ft
25. $26.63 \approx 26.66$
29. $k = a \sin\beta = 66.8 \sin 46.8° \approx 48.7$
31. $AC = 12.8$ mi **33.** 8.37 mi from A; 4.50 mi from B
35. 230 m **37.** 2.8 nautical mi **39.** 109 ft
41. $\beta = 113.4°$; $c = 10.8$ m
43. $r = 9.73$ mm, $s = 10.7$ mm **45.** 284 mi
47. 4.42×10^7 km, 2.39×10^8 km **49.** 16 cm